

Seismic analysis of an existing earth and rockfill dam subjected to the MCE by FLAC

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ABSTRACT: In recent years the height of the tailings facilities has increased significantly; also, in contrast to water dams, tailings dams are built by successive raisings, which can take place by using different materials and even different raising methods in one single dam. These characteristics could make an advanced modeling of dam seismic behavior necessary when the problem becomes complex or the consequences of failure are high, especially in seismically active areas where major earthquakes are expected. The studied tailings dam, located in the southeast of Peru, is planned to be raised up to 90m high, with the feature that some previous stages were raised by the upstream method; therefore, a portion of the embankment is currently supported by old potentially liquefiable tailings.

This paper presents the 2D dynamic analysis of an earth and rockfill dam subjected to the maximum credible earthquake (MCE) using the software Fast Lagrangian Analysis of Continua (FLAC) and the UBCSAND constitutive model for the characterization of the tailings. The main purposes of the analysis were to predict a satisfactory behavior of the dam crest, to estimate the global permanent displacements and to assess the influence of the tailings liquefaction in the seismic performance of the dam. The final results obtained after successive analyses on the dam configuration are presented here. The new raising uses the centerline method considering rockfill berms as reinforcement in the upstream slope, and flattened slopes on the crest in order to reduce damages during the earthquake.

1 INTRODUCTION

This paper presents the dynamic analysis executed in an earth and rockfill dam for a tailings facility located in southeast Peru. Currently, the dam is at an elevation of 4475masl, with a maximum height of 75m and a crest length of 342m, and will be raised until reaching the elevation of 4490masl with the centerline method whereby the dam will reach a height of 90m and its crest length will increase to approximately 440m.

The main reason of the geotechnical design was to evaluate the physical stability of the dam considering the application of seismic loads under which the dam could be subjected to in the long term. To do this, the seismic behavior of the dam was evaluated using the MCE.

The specific purposes to carry out this analysis were: (1) to verify the behavior of the crest so that its serviceability was not compromised after the design seismic event, (2) to verify the behavior of the tailings dam due to the liquefaction of the tailings disposed upstream of the centerline raising and previous stages raised by the upstream method (up to an elevation of 4460masl) and (3) to evaluate the permanent global displacement through the foundation of the dam.

The 2D dynamic analysis of the tailings dam was carried out using a FLAC numerical modeling in order to use an approximation of the performance design, which requires determining the deformations induced by the earthquake.

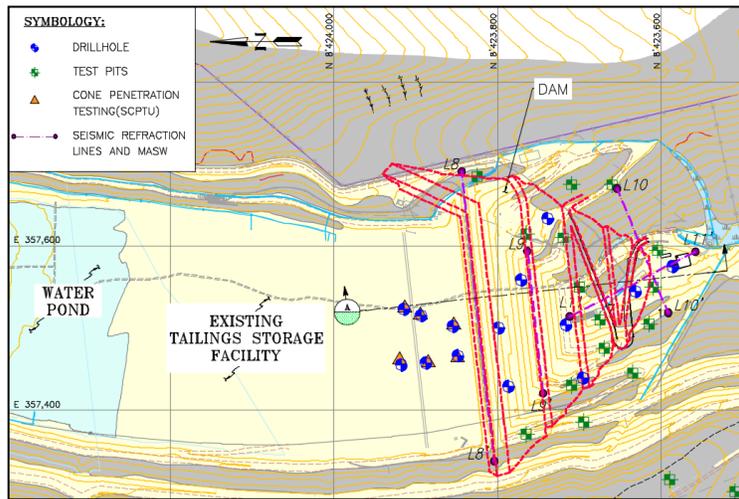


Figure 1. Plant view and the critical cross section of the tailings dam

2 GEOTECHNICAL CHARACTERIZATION

For the characterization of the properties for the relevant materials it was necessary to review and interpret the information collected from the geotechnical site investigations performed in the tailings storage facility during the previous five years. The field tests were analyzed together with the laboratory tests done on the materials of the embankment, foundation and tailings; and complemented with the data of the open pipe piezometers installed in the dam. Figure 1 shows the location of the geotechnical investigation in plant view and Figure 2 shows the critical cross section of the dam identifying the 3 main purposes of the analysis. As a hypothesis, the liquefaction of the tailings could cause a local crack inside the embankment (crack 1) and a global crack jeopardizing the dam crest (crack 2).

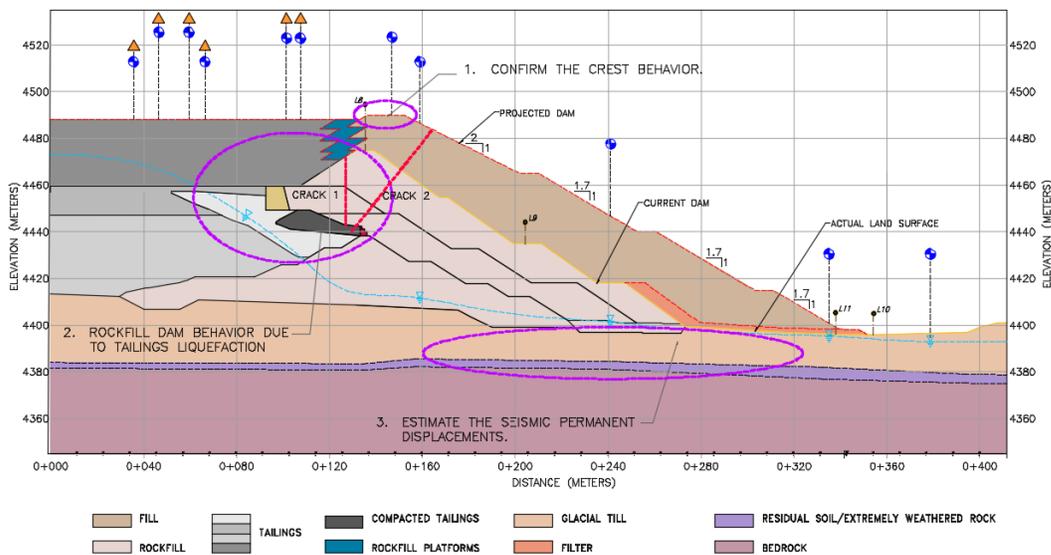


Figure 2. Geotechnical model of the critical cross section of the dam.

2.1 Foundation

The foundation of the dam can be characterized by three material types; (1) glacial deposits, (2) residual soil/extremely weathered rock and (3) bedrock conformed by sandstones and slates.

Even though the residual soil/extremely weathered rock could be a critical layer in the foundation, the depth to which it is found (between 15 to 20m) made its removal impractical; there-

fore it was important to evaluate its effects on the stability of the dam. This layer consists mainly of weathered sandstones and slates with content of dense to very dense clayey sand with gravel and firm to stiff silt/clay with low to medium plasticity. The average thickness varies from 3 to 5m in the base of the dam; likewise, the shear wave velocity values (V_s) measured by Multichannel Analysis of Surface Waves(MASW) tests, varies from 500 to 900m/s.

The geotechnical characterization of this material was conducted by using the generalized Hoek & Brown (1988) criterion for rock masses as a reference, for which the characterization obtained in the field, as well as literature parameters, were used. Then, the Mohr Coulomb envelope that best adjusts to the Hoek & Brown model for the representative confinement levels of the problem (between 400 and 2000kPa) was estimated, using a cohesion of 140kPa and a friction angle of 18°.

2.2 Embankment

Currently at the elevation 4475masl, the dam was built using compacted rockfill from mine waste and it is planned to be raised up to the elevation 4490masl using fill from nearby quarries located in glacial deposits (glacial till).

The glacial till are classified mostly as well graded gravel (GW), poorly graded gravel with clay and sand (GP-GC) and clayey sands with gravel (SC), with a plasticity index (PI) between 9 and 30, a fines content of between 2% and 48% and a maximum unit weight (obtained by the Proctor standard test) of 20kN/m³. Its strength properties have been obtained by triaxial compression tests executed in samples obtained from the quarries.

Likewise, the shear strength properties of the rockfill used on the current dam was estimated using the average rockfill envelope from Leps (1970).

2.3 Tailings.

The first step to characterize the tailings strength parameters is to determine if they have a contractive or dilative behavior during shear. Under undrained conditions contractive materials are prone to generating positive pore-water pressure during shear; however, in the case of the dilative materials, they develop negative pore pressures due to a volume increase during shear.

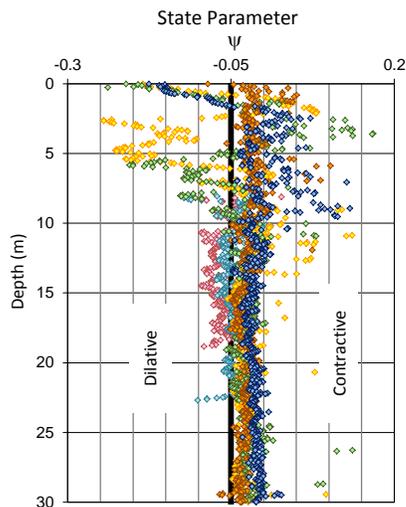


Figure 3. Tailings behavior during shearing

Figure 3 shows the evaluation of the tailings behavior based on the data obtained on the CPTu soundings, through the State Parameter (Ψ) which is defined as the difference between the existing soil void ratio (voids volume/solids volume) and the critical void ratio (boundary between the contractive and dilative behavior). Positive values of the State Parameter and even negative values higher than -0.05 indicate a contractive behavior, while negative values lower than -0.05 indicate a dilative behavior according Jefferies & Been (2006). The state parameter was estimat-

ed using the data of the CPTu and the Robertson (2010) correlation from the equivalent clean sand cone resistance $Q_{m,cs}$ suggested by Robertson & Wride (1998).

According to the data, the major portion of the tailings (loose to medium dense) will show a contractive behavior during shear, except for some surficial points (densified due to desiccation processes) which present dilative behavior. Therefore, the results indicate that it is adequate to consider the susceptibility of the tailings to the liquefaction phenomena and that is why a constitutive model of fully coupled effective stress (UBCSAND) was used to model the tailings behavior during a seismic event.

For the stability analyses, the peak undrained strength ratio (S_u/p' , where p' is the vertical effective stress) and the residual undrained shear strength ratio (S_{ur}/p') were estimated from the results of CPTu tests following established correlations between the tip resistance measured and strength parameters presented by Olson & Stark (2003) for loose materials. These correlations are based on back analysis of actual flow liquefaction failures to evaluate the shear strength mobilized at triggering of a failure.

The strength parameters of the foundation materials, embankment and tailings are presented in the Table 1 and the geotechnical model is shown in Figure 2.

Table 1. Material properties for stability analyses

Material	Unit Weight kN/m ³	Cohesion "c" (kPa)	Friction Angle (degrees)	Peak Undrained Shear Strength, Ratio S_u/p'	Residual Un- drained Shear Strength, Ratio S_{ur}/p'
Tailings	15.5-17.5	0	30	0.20-0.27	0.04-0.08
Compacted Tailings	18.5	0	35	-	-
Rockfill (current dam)	21.0	*	*	-	-
Fill (projected dam)	19.0	10	34	-	-
Glacial till	20.0	5	34	-	-
Residual soil/extremely weathered rock	21.0	140	18	-	-
Bedrock	25.0			Impenetrable	

*Shear strength is based on the average rockfill envelope from Leps (1970)

3 STABILITY ASSESSMENT

The stability analysis has been carried out using the limit equilibrium method using the Slope/W (GEO-SLOPE International) software under static, pseudo-static and post-earthquake conditions for the tailings dam so as to reach the minimum factor of safety (FoS) recommended by international guidelines. In Peru and others places, it is an accepted practice to use a minimum FoS of 1.5. The stability analysis results of the tailings dam are shown in Figure 4 and they show factors of safety higher than 1.5 under static conditions and near to 1.0 for pseudo static conditions, which is why permanent deformations are expected in the failure surface through the foundation.

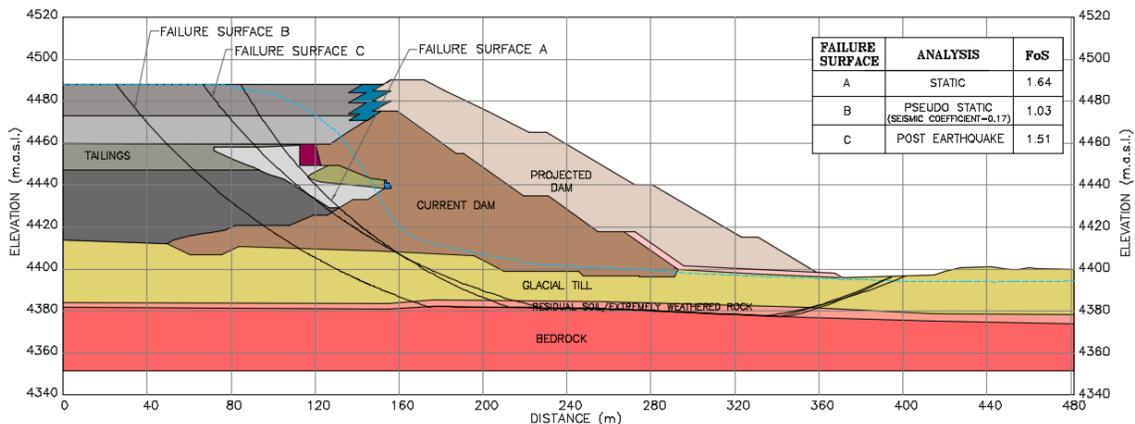


Figure 4. Results of the limit equilibrium stability analyses.

It is important to note that the global stability analysis under post-earthquake conditions considering the tailings with residual strength, does not show significant differences in comparison to the static analysis results (a decrease of FoS from 1.64 to 1.51), verifying that the tailings storage is not part of the structural zone of the dam when it comes to the global downstream stability. However, the liquefaction of the tailings could influence the slope stability upstream of the dam and could affect the behavior of the dam if the crest deformation generates a loss of containment.

4 NUMERICAL MODELING DESCRIPTION

The numerical modeling to calculate the seismic deformations of the tailings dam was performed with the 2D finite difference method using the FLAC 7.0 (Itasca International) software.

An approximation to the performance design, which requires determining the deformations induced by the earthquake, is being used more frequently in the design of large dams and large civil structures in general. Using FLAC we are capable to estimate these permanent displacements and considering them in the design, so that the structure continues to function properly after the design seismic event.

4.1 Design ground motion

Given the importance of the consequences of the dam failure, according to the CDA guide, the maximum design earthquake (MDE) recommended is the Maximum Credible Earthquake (MCE) for the site. The deterministic MCE was calculated with a peak acceleration of 0.34g (on Type B rock according to the IBC 2006) using the 84th percentile acceleration spectra, the depth and magnitude of the earthquake are 120km and 8.5Mw respectively.

Three acceleration histories of recorded earthquakes were used for the analysis: (1) Lima, 1974; (2) Moquegua, 2001 and (3) Pisco, 2007; which were adjusted to the design spectrum of the MCE using the SeismoSignal and SeismoMachth softwares. These accelerograms were conveniently edited, considering only the most intense portion of the shaking, in order to optimize the numerical calculation as shown in Figure 5. They were also applied as a function of the shear strain based on the velocity history on the quiet base boundary of the numerical model to adequately represent the application of the seismic shaking.

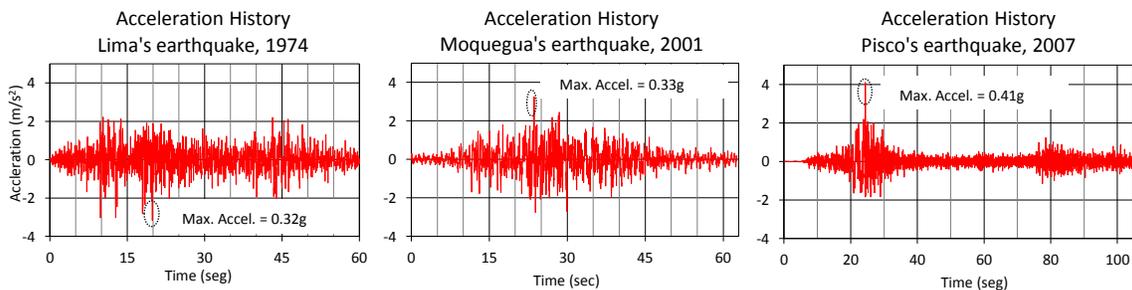


Figure 5. Acceleration histories adjusted to the design spectra.

4.2 Grid and model

The size of the model was developed taking into account that: 1) the critical failure surface can be developed properly; 2) lateral boundary conditions have no significant influence on the model; and 3) the depth of the base allows proper propagation of the seismic signal and prevents the model rotation during the simulation. The element size was selected primarily considering an adequate seismic wave propagation according to Kuhlemeyer & Lysmer (1973) considerations, related to the size of the element and the material stiffness.

The final configuration of the tailings dam was developed modeling three intermediate stages for the estimation of the initial state prior to the dynamic analysis: a) the tailings dam built to an elevation of 4460masl, b) the raising to an elevation of 4475masl and c) the raising to an eleva-

tion of 4490masl. The various stages were solved considering the flow and mechanical conditions of the structure to determine the initial stress state. Flux analyses were performed considering the operational conditions and the readings of open pipe piezometers installed in the area of the tailings dam. Mechanical analyzes were performed considering drained conditions of various materials and linear elastic modulus, which was considered as a fraction of the small strains modulus (G_{max} and K_{max}). The initial state of pore-water pressures is shown in Figure 6.

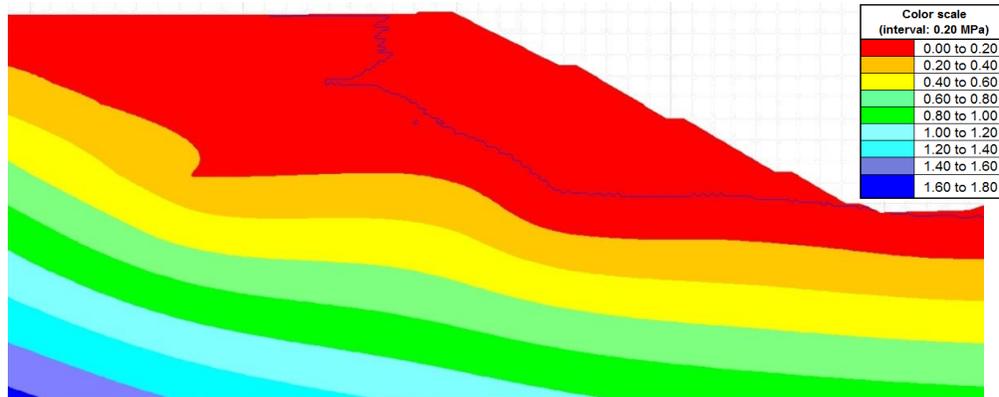


Figure 6. Initial Pore-water pressure prior to shaking

4.3 Dynamic properties and damping for non-liquefiable materials

The shear and bulk modulus were estimated from the V_s measurements obtained from the MASW tests and then applied to the model with an effective confinement stress dependency. Therefore, due to the height of the dam, the confinement variation creates a nonuniform condition of stiffness in the embankment and the foundation. The stiffness profile was estimated using the following equations:

$$G_{max} = 22 \times K_{2max} \times \sqrt{(\sigma'_m \times P_{atm})} \quad \text{and} \quad G_{max} = \rho \times V_s^2$$

Where G_{max} is the maximum shear modulus, K_{2max} is the shear modulus coefficient, σ'_m is the mean confinement stress, P_{atm} is the atmospheric pressure, ρ is the density and V_s is the shear wave velocity.

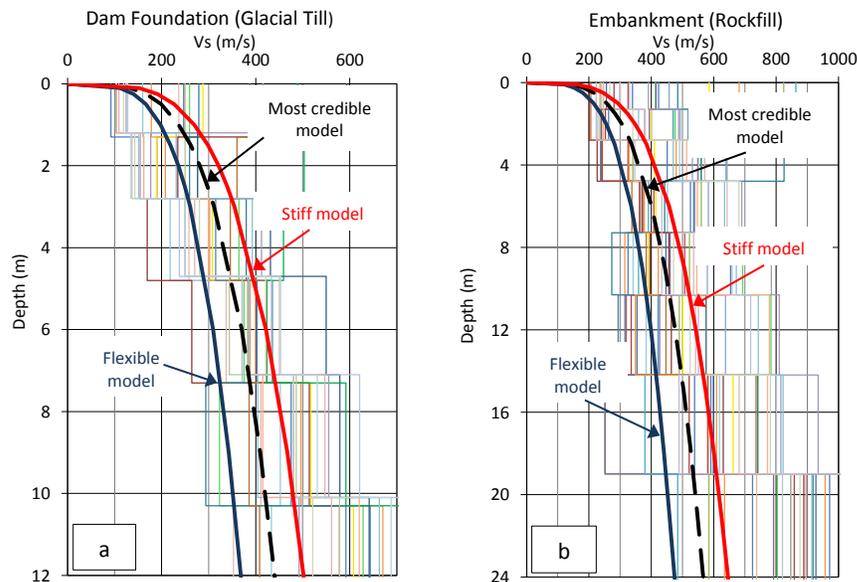


Figure 7. Shear wave velocity profile of the glacial till (a) and the rockfill (b) within the MASW data.

The data of the MASW tests and fitting curves for velocity profiles of shear waves (V_s) are shown in Figure 7. As can be seen, the measured field values were very variable, so the numerical modeling considered analysis with variable stiffness, as follows: (1) flexible model, closer to conservative literature values, (2) stiff model, closer to the mean of the measured field values and (3) most credible model, which is an intermediate between the 2 previous models.

A shear modulus profile for the dam was developed with these considerations as shown in Figure 8.

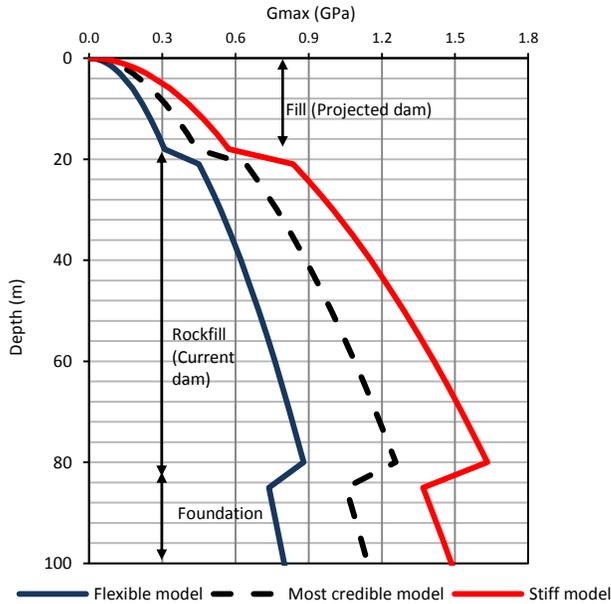


Figure 8. Shear modulus profile of the dam.

Also, it was important to assign appropriate damping properties to the materials using the FLAC damping models. A short amount of Rayleigh damping of 0.2% was assigned throughout the model (with a central frequency of 0.5Hz, the dominate frequency on the velocity history of the design seismic motion) only to remove high frequency noise in the running.

The Mohr-Coulomb model with hysteretic damping was used for non-liquefiable materials (foundation and embankment). Hysteretic damping allows to simulate damping variations in space and time because it is directly associated to the degree of shear strain that each element has. This damping was assigned according to literature, using the reduction curves of dynamic shear modulus and damping ratio based on Seed et al. (1986) for gravels. For this purpose, the sigmoidal model "sig3" of hysteretic damping in FLAC was calibrated simulating these theoretical curves, as seen in Figure 9.

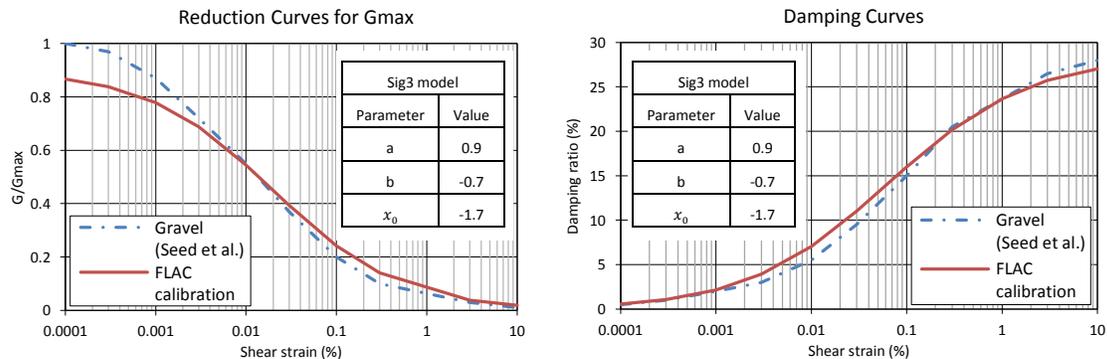


Figure 9. Cyclic FLAC simulations for sig3 model vs Seed curve on non-liquefiable materials

4.4 Dynamic properties of liquefiable materials (tailings)

The tailings were considered materials prone to liquefaction (as seen in Figure 3); therefore their dynamic behavior was modeled using the UBCSAND constitutive model, which has the capability to model the strength and stiffness degradation and pore-water pressure development during the shaking. The UBCSAND model was calibrated by simulating a cyclic shear test in FLAC and the results were compared with the Cyclic Direct Simple Shear (CDSS) laboratory tests as shown in Figure 10.

While the weighting curve (CRR vs number of cycles to begin liquefaction) in the tailings is flatter than expected, the $(N_1)_{60}$ value equal to 8 for the tailings zone tested was used obtaining a representative behavior of the laboratory tests. Other less dense tailings zones were modeled using $(N_1)_{60}$ between 5 and 7 based on estimates from the CPTu and SPT tests. To consider the effect of the confinement on the seismic behavior of the tailings, the $K\sigma$ factor recommended by Youd et al. (2001) was implicitly used on the m_hfac1 adjusting factor. These adjusting factors are shown in Table 2.

The UBCSAND model includes a hysteretic damping in its formulation, so it was not necessary to assign additional damping.

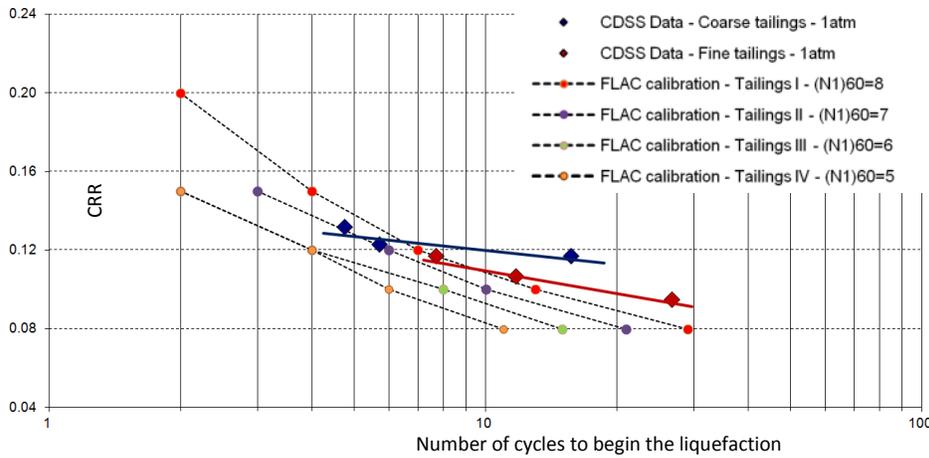


Figure 10. Calibration of the UBCSAND model for 1 atm with CDSS test data.

Table 2. The m_hfac adjusting factors for tailings.

Confinement pressure (kPa)	m_hfac1^*
<100	1.00
100 - 300	0.68
300 - 700	0.41
>700	0.30

* assuming an average relative density of 40%

5 NUMERICAL MODELING RESULTS

In this section are presented the final results after the several analyses performed, which includes the 3 acceleration histories, reversals on the earthquake orientation and the variation in the model stiffness. In general, the more damaging earthquake during the simulations was to Lima 1974 earthquake (adjusted to the MCE) and their results are presented below.

5.1 Crest behavior

A series of previous analyses were developed as part of the evaluation of the dam seismic response in order to optimize the geometry of the dam without affecting its structural integrity. Certain indicators were considered to be evaluated for a proper final configuration, such as: (1) the crest settlement; and (2) the shear strains in the slopes.

The preliminary results showed that it was possible that settlements occur on the crest on the order of 1m (upstream slope) and the shear deformations downstream (near 10%) and upstream

(even greater than 20%) were interpreted as a condition that could trigger failure mechanisms, compromising the physical stability of the dam crest. Based on these results, the geometry of the tailings dam was modified seeking to reduce these shear deformations and thus limit the possibility of occurrence of failures on the crest slopes. The results of these dynamic analyses for the preliminary and final crest configurations are shown in in Table 3 and Figure 11.

The change on the configuration of the crest consists of flattening the downstream crest slope from 1.7H:1V to 2H:1V and the placement of rockfill platforms on the upstream slope, in order to improve the crest behavior during the seismic event. With this final configuration, the seismic behavior was evaluated as adequate, reducing the possibility that the tailings contents are discharged to the outside.

Table 3. Shear strain on the crest on preliminary and final configurations.

	Preliminar configuration		Final configuration	
	Shear Strain	Settlement (m)	Shear Strain	Settlement (m)
Upstream slope	>20%	1.0	<1%	<0.1
Downstream slope	9%	0.4	5%	0.2

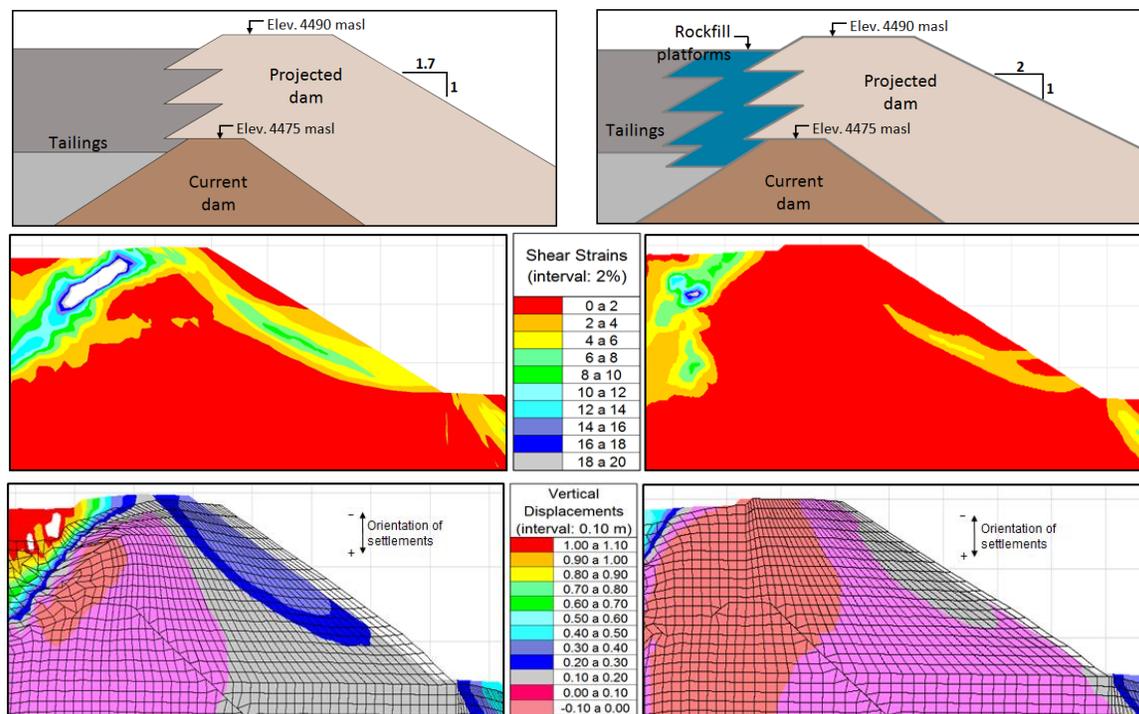


Figure 11. Dynamic analysis results for preliminary and final crest configurations

5.2 Permanent horizontal displacement through the foundation of the dam

As a screening procedure, the permanent deformations through the foundation of the dam were estimated using the Bray & Travasarou (2007) method based on a fully coupled stick-slip sliding block model to simulate the dynamic behavior of dams as shown in Table 4.

Table 4. Simplified displacements results (Bray & Travasarou 2007) through the foundation of the dam

	Ts*	Sa (1.5Ts)	Displacement**
	sec	g	cm
Flexible model	0.50	0.40	2.2 – 13.0
Most credible model	0.42	0.43	2.9 – 14.0
Stiff Model	0.37	0.47	3.9 – 16.4

*Calculated as 2.6H/Vs

**P84 – P16 range of probability of exceedance

Several FLAC analyses were performed including reversal earthquake orientation and the flexible, stiff and most credible model, concluding a range of horizontal permanent deformation of 5.0 to 30.0cm as shown in Table 5. These displacements are slightly higher than the simplified deformation results, due to the fact that the design ground motion is an 8.5Mw subduction earthquake, different to the database used on the calibration of the simplified method (crustal earthquakes with magnitudes between 5.5 and 7.6). Also, these displacements are considered not critical for the operation of the dam.

Table 5. Horizontal displacements through the foundation of the dam by FLAC analyses

	Lima 1974	Moquegua 2001	Pisco 2007
Flexible model	20 - 25cm	20 - 25cm	10 – 15cm
Most credible model	20 - 25cm	25 - 30cm	10 – 15cm
Stiff Model	20 - 25cm	15 - 20cm	5 – 10cm

A representative behavior of the horizontal displacements in the dam is shown in Figure 12 with values up to 1.0m in the intermediate benches.

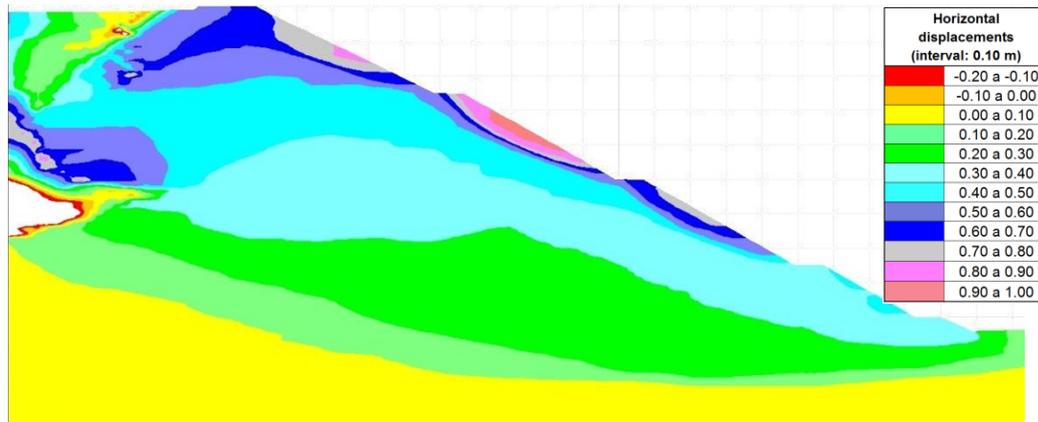


Figure 12. Horizontal displacements after the seismic event.

5.3 Tailings liquefaction effects

The analyzes show that the liquefaction observed in the area of the tailings do not adversely affect the structural integrity of the dam (after the modification of the crest configuration), because no significant global deformations or shear stresses are observed to this area, however, vertical deformations of up to 0.5 m are observed in the area of the embankment supported by tailings (built during previous upstream raisings) as shown in Figure 13. These deformations could cause local cracks (crack 1) which do not propagate to the crest affecting the serviceability of the dam (crack 2).

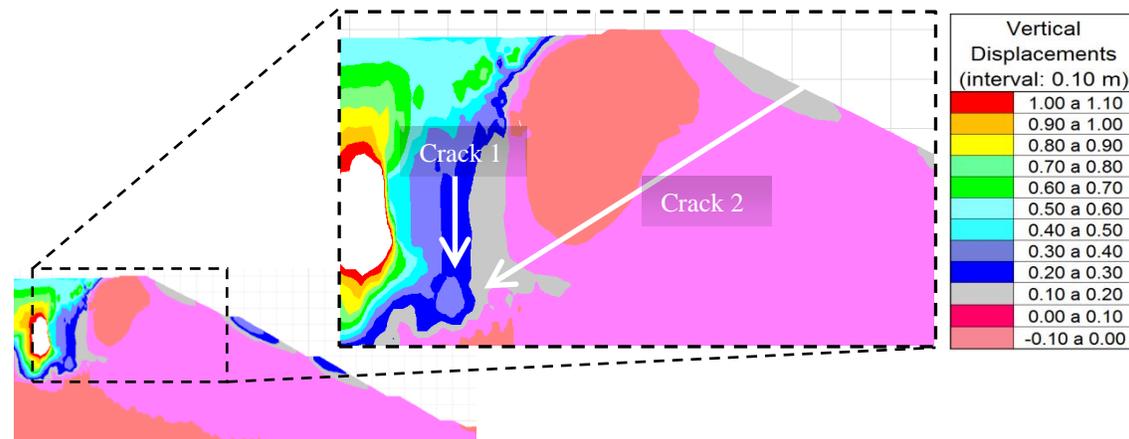


Figure 13. Vertical displacements after the seismic event.

In Figure 14 is shown the development of pore-water pressure ratio R_u in the tailings during the earthquake of Lima (1974). Some of the surficial areas developed excess pore-water pressure faster than the areas located deeper and less contractive. In general, all the tailings were liquefied (R_u above 0.7) before the first 15 seconds of shaking with the exception of the compacted tailings which have not reached liquefaction. This contractive behavior observed is consistent with the results obtained in the SCPTu tests.

Also, a post-earthquake analysis was performed in FLAC assigning liquefied strengths and a reduced elastic shear stiffness equal to 10 times the liquefied shear strength in all the elements that had exceeded the R_u value of 0.7. In this verification, no significant additional deformations were obtained, so it is concluded that the dynamic analysis itself achieved representative degraded properties of the tailings.

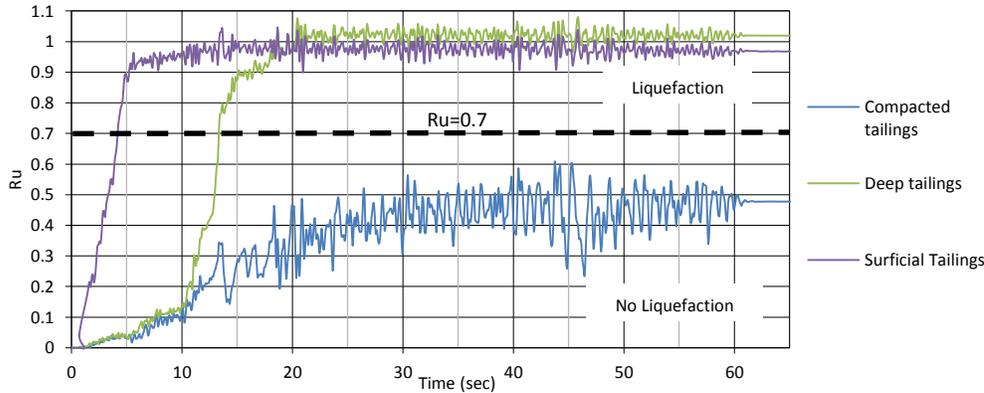


Figure 14. Pore-water pressure ratio development in tailings

6 CONCLUSIONS

It is observed that the numerical modeling for the dynamic analysis of the dam provides useful insight into the seismic performance of the dam, from which it can be concluded that geotechnical design objectives were met.

Due to the variability of the geotechnical parameters and uncertainties in the model, several analyses are recommended using some sensitivity of the main parameters and different seismic accelerations histories for more valuable results.

According to the results of dynamic analysis at the crest of the dam, it is observed that the geometry of the crest is appropriate, minor deformations are expected and the containment function of the dam is preserved.

It was verified that the influence of the liquefaction of the tailings, initially considered critical, would not affect the structural integrity of the tailings dam, since no significant global shear stresses deformation or through the embankment is observed to this area.

Also, the permanent horizontal deformation through the foundation of the dam were estimated which are slightly higher than those estimated by simplified methods, but still low enough for a proper operation of the dam. Simplified procedures could have limitations for dynamic analysis using intense subduction earthquakes due to the different database nature with which the methods were calibrated.

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